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# Improved Training of Wasserstein GANs

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## Abstract

Generative Adversarial Networks (GANs) are powerful generative models, but suffer from training instability. The recently proposed Wasserstein GAN (WGAN) makes progress toward stable training of GANs, but can still generate low-quality samples or fail to converge in some settings. We find that these problems are often due to the use of weight clipping in WGAN to enforce a Lipschitz constraint on the critic, which can lead to pathological behavior. We propose an alternative to clipping weights: penalize the norm of gradient of the critic with respect to its input. Our proposed method performs better than standard WGAN and enables stable training of a wide variety of GAN architectures with almost no hyperparameter tuning, including 101-layer ResNets and language models over discrete data. We also achieve high quality generations on CIFAR-10 and LSUN bedrooms. <sup>1</sup>

## 1 Introduction

Generative Adversarial Networks (GANs) [9] are a powerful class of generative models that cast generative modeling as a game between two networks: a generator network produces synthetic data given some noise source and a discriminator network discriminates between the generator's output and true data. GANs can produce very visually appealing samples, but are often hard to train, and much of the recent work on the subject [22, 18, 2, 20] has been devoted to finding ways of stabilizing training. Despite this, consistently stable training of GANs remains an open problem.

In particular, [1] provides an analysis of the convergence properties of the value function being optimized by GANs. Their proposed alternative, named Wasserstein GAN (WGAN) [2], leverages the Wasserstein distance to produce a value function which has better theoretical properties than the original. WGAN requires that the discriminator (called the *critic* in that work) must lie within the space of 1-Lipschitz functions, which the authors enforce through weight clipping.

Our contributions are as follows:

1. On toy datasets, we show how critic weight clipping can lead to pathological behavior.
2. We propose *WGAN with gradient penalty*, which does not suffer from the same issues.
3. We demonstrate stable training of many difficult GAN architectures with default settings, performance improvements over weight clipping, high-quality generations on CIFAR-10 and LSUN bedrooms, and a character-level GAN language model with a very simple architecture.

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<sup>1</sup>Code for our models is available at [https://github.com/igul222/improved\\_wgan\\_training](https://github.com/igul222/improved_wgan_training).

## 2 Background

### 2.1 Generative adversarial networks

The GAN training strategy is to define a game between two competing networks. The *generator* network maps a source of noise to the input space. The *discriminator* network receives either a generated sample or a true data sample and must distinguish between the two. The generator is trained to fool the discriminator.

Formally, the game between the generator  $G$  and the discriminator  $D$  is the minimax objective:

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim \mathbb{P}_r} [\log(D(\mathbf{x}))] + \mathbb{E}_{\tilde{\mathbf{x}} \sim \mathbb{P}_g} [\log(1 - D(\tilde{\mathbf{x}}))], \quad (1)$$

where  $\mathbb{P}_r$  is the data distribution and  $\mathbb{P}_g$  is the model distribution implicitly defined by  $\tilde{\mathbf{x}} = G(\mathbf{z})$ ,  $\mathbf{z} \sim p(\mathbf{z})$  (the input  $\mathbf{z}$  to the generator is sampled from some simple noise distribution, such as the uniform distribution or a spherical Gaussian distribution).

If the discriminator is trained to optimality before each generator parameter update, then minimizing the value function amounts to minimizing the Jensen-Shannon divergence between the data and model distributions on  $\mathbf{x}$  [9], but doing so often leads to vanishing gradients as the discriminator saturates. In practice, [9] advocates that the generator be instead trained to maximize  $\mathbb{E}_{\tilde{\mathbf{x}} \sim \mathbb{P}_g} [\log(D(\tilde{\mathbf{x}}))]$ , which goes some way to circumvent this difficulty. However, even this modified loss function can misbehave in the presence of a good discriminator [1].

### 2.2 Wasserstein GANs

[2] argues that the divergences which GANs typically minimize are potentially not continuous with respect to the generator’s parameters, leading to training difficulty. They propose instead using the *Earth-Mover* (also called Wasserstein-1) distance  $W(q, p)$ , which is informally defined as the minimum cost of transporting mass in order to transform the distribution  $q$  into the distribution  $p$  (where the cost is mass times transport distance). Under mild assumptions,  $W(q, p)$  is continuous everywhere and differentiable almost everywhere.

The WGAN value function is constructed using the Kantorovich-Rubinstein duality [24] to obtain

$$\min_G \max_{D \in \mathcal{D}} \mathbb{E}_{\mathbf{x} \sim \mathbb{P}_r} [D(\mathbf{x})] - \mathbb{E}_{\tilde{\mathbf{x}} \sim \mathbb{P}_g} [D(\tilde{\mathbf{x}})] \quad (2)$$

where  $\mathcal{D}$  is the set of 1-Lipschitz functions and  $\mathbb{P}_g$  is once again the model distribution implicitly defined by  $\tilde{\mathbf{x}} = G(\mathbf{z})$ ,  $\mathbf{z} \sim p(\mathbf{z})$ . In that case, under an optimal discriminator (called a *critic* in the paper, since it’s not trained to classify), minimizing the value function with respect to the generator parameters minimizes  $W(\mathbb{P}_r, \mathbb{P}_g)$ .

The WGAN value function results in a critic function whose gradient with respect to its input is better behaved than its GAN counterpart, making optimization of the generator easier. Additionally, WGAN has the desirable property that its value function correlates with sample quality, which is not the case for GANs.

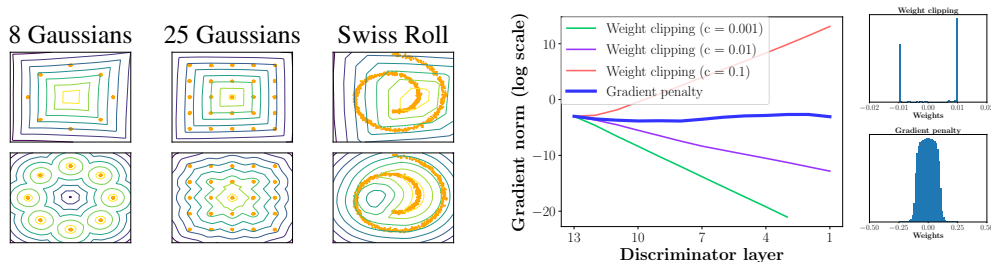
To enforce the Lipschitz constraint on the critic, [2] propose to clip the weights of the critic to lie within a compact space  $[-c, c]$ . The set of functions satisfying this constraint is a subset of the  $k$ -Lipschitz functions for some  $k$  which depends on  $c$  and the critic architecture. In the following sections, we demonstrate some of the issues with this approach and propose an alternative.

### 2.3 Properties of the optimal WGAN critic

In order to understand why weight clipping is problematic in a WGAN critic, as well as to motivate our approach, we highlight some properties of the optimal critic in the WGAN framework. We more formally state and prove these in the Appendix.

If the optimal critic under the Kantorovich-Rubinstein dual  $D^*$  is differentiable, and  $x$  is a point from our generator distribution  $\mathbb{P}_g$ , then there is a point  $y$  sampled from the true distribution  $\mathbb{P}_r$  such that the gradient of  $D^*$  at all points  $x_t = (1 - t)x + ty$  on a straight line between  $x$  and  $y$  points directly towards  $y$ , meaning  $\nabla D^*(x_t) = \frac{y - x_t}{\|y - x_t\|}$ .

*This implies that the optimal critic has unit gradient norm almost everywhere under  $\mathbb{P}_r$  and  $\mathbb{P}_g$ .*



(a) Value surfaces of WGAN critics trained to optimality on toy datasets using (top) weight clipping and (bottom) gradient penalty. Critics trained with weight clipping fail to capture higher moments of the data distribution. The ‘generator’ is held fixed at the real data plus Gaussian noise.

(b) (left) Gradient norms of deep WGAN critics during training on toy datasets either explode or vanish when using weight clipping, but not when using a gradient penalty. (right) Weight clipping (top) pushes weights towards two values (the extremes of the clipping range), unlike gradient penalty (bottom).

Figure 1: Gradient penalty in WGANs does not exhibit undesired behavior like weight clipping.

### 3 Difficulties with weight constraints

We find that weight clipping in WGAN leads to optimization difficulties, and that even when optimization succeeds the resulting critic can have a pathological value surface. We explain these problems below and demonstrate their effects; however we do not claim that each one always occurs in practice, nor that they are the only such mechanisms.

Our experiments use the specific form of weight constraint from [2] (hard clipping of the magnitude of each weight), but we also tried other weight constraints (L2 norm clipping, weight normalization), as well as soft constraints (L1 and L2 weight decay) and found that they exhibit similar problems.

To some extent these problems can be mitigated with batch normalization in the critic, which [2] use in all of their experiments. However even with batch normalization, we observe that very deep WGAN critics often get stuck in a bad regime and fail to learn.

#### 3.1 Capacity underuse

Implementing a  $k$ -Lipshitz constraint via weight clipping biases the critic towards much simpler functions. As stated previously, the optimal critic under the WGAN loss function has unit gradient norm almost everywhere; under a weight-clipping constraint, most neural network architectures can only attain their maximum gradient norm of  $k$  when they learn extremely simple functions.

To demonstrate this, we train WGAN critics with weight clipping to optimality on several toy distributions, holding the generator distribution  $\mathbb{P}_g$  fixed at the real distribution plus unit-variance Gaussian noise. We plot value surfaces of the critics in Figure 1a. We omit batch normalization in the critic. In each case, the critic trained with weight clipping ignores higher moments of the data distribution and instead models very simple approximations to the optimal functions. In contrast, our approach does not suffer from this behavior.

#### 3.2 Exploding and vanishing gradients

We observe that the WGAN optimization process is difficult because of interactions between the weight constraint and the cost function, which inevitably result in either vanishing or exploding gradients, depending on the value of the clipping threshold  $c$ .

To demonstrate this, we train WGAN on the Swiss Roll toy dataset, varying the clipping threshold  $c$  in  $[10^{-1}, 10^{-2}, 10^{-3}]$ , and plot the norm of the gradient of the critic loss with respect to successive layers of activations. Both generator and critic are 12-layer ReLU MLPs without batch normalization. Figure 1b shows that for each of these values, the gradient either grows or decays exponentially as we move farther back in the network. We find our method results in more stable gradients that neither vanish nor explode, allowing training of more complicated networks.

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**Algorithm 1** WGAN with gradient penalty. We use default values of  $\lambda = 10$ ,  $n_{\text{critic}} = 5$ ,  $\alpha = 0.0001$ ,  $\beta_1 = 0$ ,  $\beta_2 = 0.9$ .

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**Require:** The gradient penalty coefficient  $\lambda$ , the number of critic iterations per generator iteration  $n_{\text{critic}}$ , the batch size  $m$ , Adam hyperparameters  $\alpha, \beta_1, \beta_2$ .

**Require:** initial critic parameters  $w_0$ , initial generator parameters  $\theta_0$ .

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1: while  $\theta$  has not converged do
2:   for  $t = 1, \dots, n_{\text{critic}}$  do
3:     for  $i = 1, \dots, m$  do
4:       Sample real data  $\mathbf{x} \sim \mathbb{P}_r$ , latent variable  $\mathbf{z} \sim p(\mathbf{z})$ , a random number  $\epsilon \sim U[0, 1]$ .
5:        $\tilde{\mathbf{x}} \leftarrow G_\theta(\mathbf{z})$ 
6:        $\hat{\mathbf{x}} \leftarrow \epsilon \mathbf{x} + (1 - \epsilon) \tilde{\mathbf{x}}$ 
7:        $L^{(i)} \leftarrow D_w(\tilde{\mathbf{x}}) - D_w(\mathbf{x}) + \lambda (\|\nabla_{\hat{\mathbf{x}}} D_w(\hat{\mathbf{x}})\|_2 - 1)^2$ 
8:     end for
9:      $w \leftarrow \text{Adam}(\nabla_w \frac{1}{m} \sum_{i=1}^m L^{(i)}, w, \alpha, \beta_1, \beta_2)$ 
10:  end for
11:  Sample a batch of latent variables  $\{\mathbf{z}^{(i)}\}_{i=1}^m \sim p(\mathbf{z})$ .
12:   $\theta \leftarrow \text{Adam}(\nabla_\theta \frac{1}{m} \sum_{i=1}^m -D_w(G_\theta(\mathbf{z})), \theta, \alpha, \beta_1, \beta_2)$ 
13: end while

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## 4 Gradient penalty

We now propose an alternative way to enforce the Lipschitz constraint. A differentiable function is 1-Lipschitz if and only if it has gradients with norm at most 1 everywhere, so we consider directly constraining the gradient norm of the critic’s output with respect to its input. To circumvent tractability issues, we enforce a soft version of the constraint with a penalty on the gradient norm for random samples  $\hat{\mathbf{x}} \sim \mathbb{P}_{\hat{\mathbf{x}}}$ . Our new objective is

$$L = \underbrace{\mathbb{E}_{\hat{\mathbf{x}} \sim \mathbb{P}_g} [D(\hat{\mathbf{x}})] - \mathbb{E}_{\mathbf{x} \sim \mathbb{P}_r} [D(\mathbf{x})]}_{\text{Original critic loss}} + \lambda \underbrace{\mathbb{E}_{\hat{\mathbf{x}} \sim \mathbb{P}_{\hat{\mathbf{x}}}} [(\|\nabla_{\hat{\mathbf{x}}} D(\hat{\mathbf{x}})\|_2 - 1)^2]}_{\text{Our gradient penalty}}. \quad (3)$$

**Sampling distribution** We implicitly define  $\mathbb{P}_{\hat{\mathbf{x}}}$  sampling uniformly along straight lines between pairs of points sampled from the data distribution  $\mathbb{P}_r$  and the generator distribution  $\mathbb{P}_g$ . This is motivated by the fact that the graph of the optimal critic consists of straight lines connecting points from  $\mathbb{P}_r$  and  $\mathbb{P}_g$  (see [subsection 2.3](#)). Given that enforcing the unit gradient norm constraint everywhere is intractable, enforcing it only along these straight lines seems sufficient and experimentally results in good performance.

**Penalty coefficient** All experiments in this paper use  $\lambda = 10$ , which we found to work well across a variety of architectures and datasets ranging from toy tasks to large ImageNet CNNs.

**No critic batch normalization** Most prior GAN implementations [21, 22, 2] use batch normalization in both the generator and the discriminator to help stabilize training, but batch normalization changes the form of the discriminator’s problem from mapping a single input to a single output to mapping from an entire batch of inputs to a batch of outputs [22]. Our penalized training objective is no longer valid in this setting, since we penalize the norm of the critic’s gradient with respect to each input independently, and not the entire batch. To resolve this, we simply omit batch normalization in the critic in our models, finding that they perform well without it.

Our method works with normalization schemes which don’t introduce correlations between examples. In particular, we recommend layer normalization [3] as a replacement for batch normalization.

**Two-sided penalty** We encourage the norm of the gradient to go towards 1 (two-sided penalty) instead of just staying below 1 (one-sided penalty). In practice, we found this to converge slightly faster and to better optima. Empirically this seems not to constrain the critic too much, likely because the optimal WGAN critic anyway has gradients with norm 1 almost everywhere under  $\mathbb{P}_r$  and  $\mathbb{P}_g$  and in large portions of the region in between (see [subsection 2.3](#)).

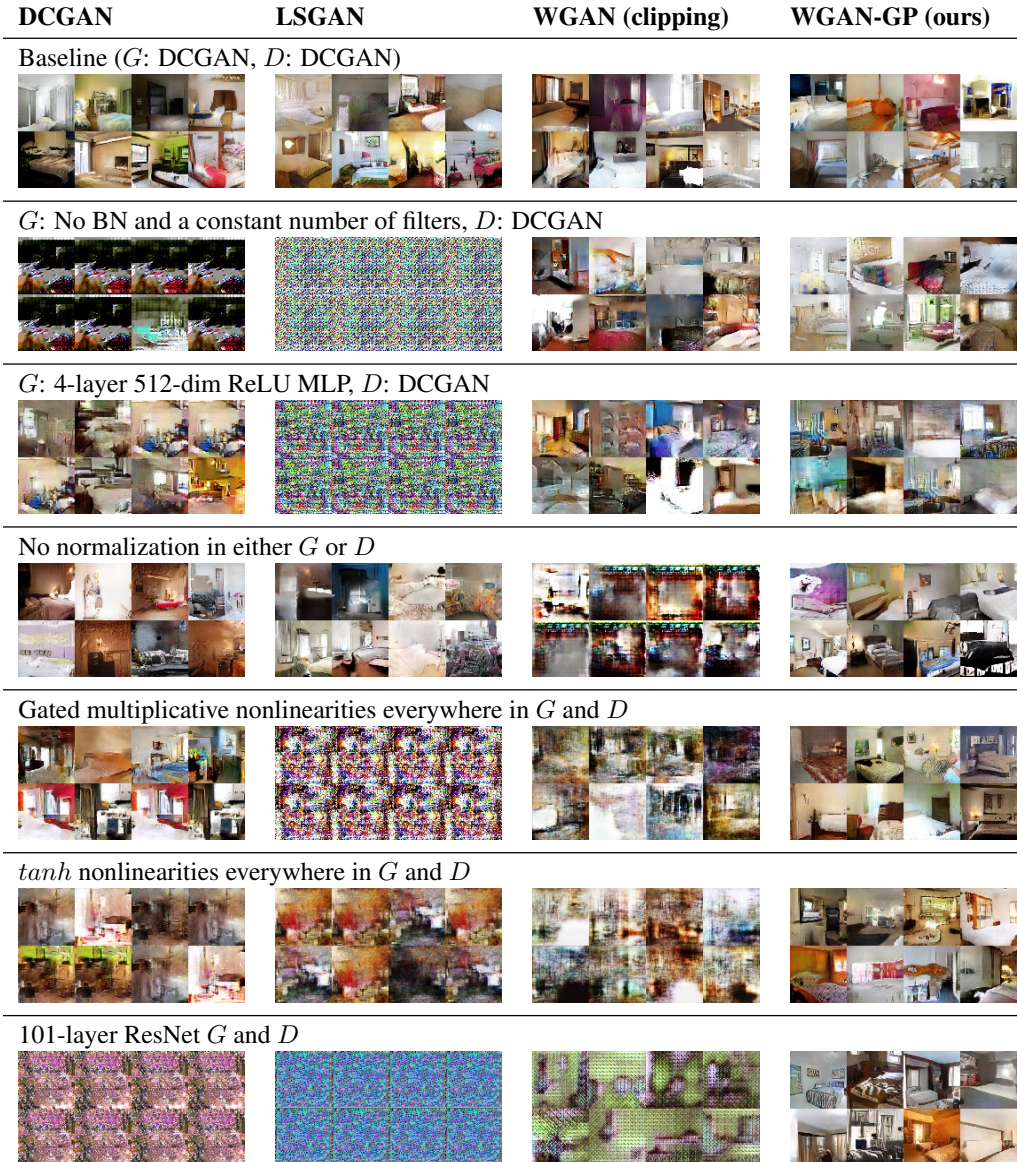


Figure 2: Difficult GAN architectures trained with different methods. Only WGAN with gradient penalty succeeds in training every architecture with a single choice of default settings.

## 5 Experiments

### 5.1 Architecture robustness

To demonstrate the stability of our method’s training process, we train a wide variety of GAN architectures on the LSUN bedrooms dataset [30]. In addition to the baseline DCGAN architecture from [21], we choose six architectures which we think are difficult to train: (1) no BN and a constant number of filters in the generator, as in [2], (2) 4-layer 512-dim ReLU MLP generator, as in [2], (3) no normalization in either the discriminator or generator (4) gated multiplicative nonlinearities, as in [23], (5)  $\tanh$  nonlinearities, and (6) 101-layer ResNet generator and discriminator.

Although we do not claim it is impossible without our method, to the best of our knowledge this is the first time very deep residual networks were successfully trained in a GAN setting. For each architecture, we train models using four different GAN procedures: WGAN with gradient penalty,

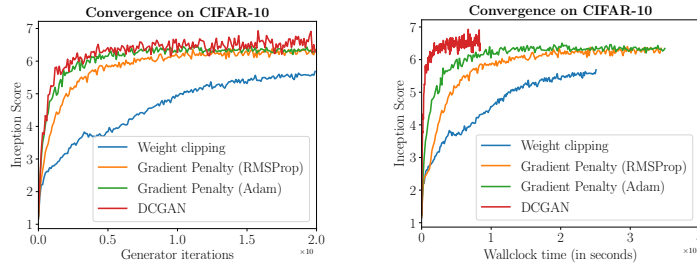


Figure 3: CIFAR-10 Inception score over generator iterations (left) or wall-clock time (right) for four models: WGAN with weight clipping, WGAN with gradient penalty and RMSProp (to control for the optimizer), WGAN with gradient penalty and Adam, and DCGAN. Even with the same learning rate, gradient penalty significantly outperforms weight clipping. DCGAN converges faster, but WGAN with gradient penalty achieves similar scores with improved stability.

WGAN with weight clipping, DCGAN [21], and Least-Squares GAN [17]. For each objective, we used the default set of optimizer hyperparameters recommended in that work (except LSGAN, where we searched over learning rates).

For WGAN with gradient penalty, we replace any batch normalization in the discriminator with layer normalization (see section 4). We train each model for 200K iterations and present samples in Figure 2. WGAN with gradient penalty is the only method which successfully trains every architecture with a single set of default hyperparameters. For every other training method, some of these architectures are unstable or suffer from mode collapse.

## 5.2 Improved performance over weight clipping

One advantage of our method over weight clipping is improved training speed and sample quality. To demonstrate this, we train WGANs with weight clipping and our gradient penalty on CIFAR-10 [13] and plot Inception scores [22] over the course of training (see Figure 3). For WGAN with gradient penalty, we train one model with the same optimizer (RMSProp) and learning rate as WGAN with weight clipping, and another model with Adam and a higher learning rate. Even with the same optimizer, our method converges faster and to a better final score than using weight clipping. Using Adam further improves performance. We also plot the performance of DCGAN [21] and observe that although our method converges more slowly (in terms of wall-clock time) than DCGAN, its performance is more stable at convergence.

## 5.3 Sample quality on CIFAR-10 and LSUN bedrooms

For equivalent architectures, our method achieves comparable sample quality to the standard GAN objective. However the increased stability allows us to improve sample quality by exploring a wider range of architectures. To demonstrate this, we find an architecture which establishes a new state of the art Inception score on unsupervised CIFAR-10 (Figure 4). When we add label information (using the procedure in [19]), the same architecture outperforms all other published models except for SGAN.

We also train a deep ResNet on  $128 \times 128$  LSUN bedrooms and show samples in Figure 6. We believe these samples are at least competitive with the best reported so far on any resolution for this dataset.

## 5.4 Character-level language modeling

Using GANs to model language is challenging, largely because text is a sequence of discrete tokens and discrete output units in the generator are difficult to backpropagate through. Most past attempts have addressed this with the REINFORCE gradient estimator [27] or continuous approximations to discrete sampling [12, 16], but have seen limited success without resorting to pretraining or joint training with a typical supervised maximum-likelihood objective [31, 14, 29, 5, 15]. [10] also propose a discrete GAN training method but do not evaluate on language modeling.

Method	Score	Method	Score
ALI [8] (in [26])	5.34 ± .05	SteinGAN [25]	6.35
BEGAN [4]	5.62	DCGAN (with labels, in [25])	6.58
DCGAN [21] (in [11])	6.16 ± .07	Improved GAN [22]	8.09 ± .07
Improved GAN (-L+HA) [22]	6.86 ± .06	AC-GAN [19]	8.25 ± .07
EGAN-Ent-VI [7]	7.07 ± .10	SGAN-no-joint [11]	8.37 ± .08
DFM [26]	7.72 ± .13	WGAN-GP ResNet (ours)	8.42 ± .10
<b>WGAN-GP ResNet (ours)</b>	<b>7.86 ± .07</b>	<b>SGAN [11]</b>	<b>8.59 ± .12</b>

Unsupervised Inception scores

Supervised Inception scores

Figure 4: Inception scores on CIFAR-10. Among unsupervised models, our model achieves state-of-the-art performance. With the addition of label information (following [19]) and no other tuning, our model outperforms all other supervised architectures except SGAN.



Figure 5: (left) CIFAR-10 samples generated by our unsupervised model. (right) Conditional CIFAR-10 samples, from adding AC-GAN conditioning to our unconditional model. Samples from the same class are displayed in the same column.

Table 1: Top: Samples from a WGAN character-level language model trained with our method on sentences from the Billion Word dataset, truncated to 32 characters. The model learns to directly output one-hot character embeddings from a latent vector without any discrete sampling step. Bottom: Samples from the same architecture trained with the standard GAN objective. Better results with the standard GAN objective are possible, but require either tuning or more complicated architectures.

<b>WGAN with gradient penalty (1D CNN)</b>	
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<b>Standard GAN objective (same architecture)</b>	
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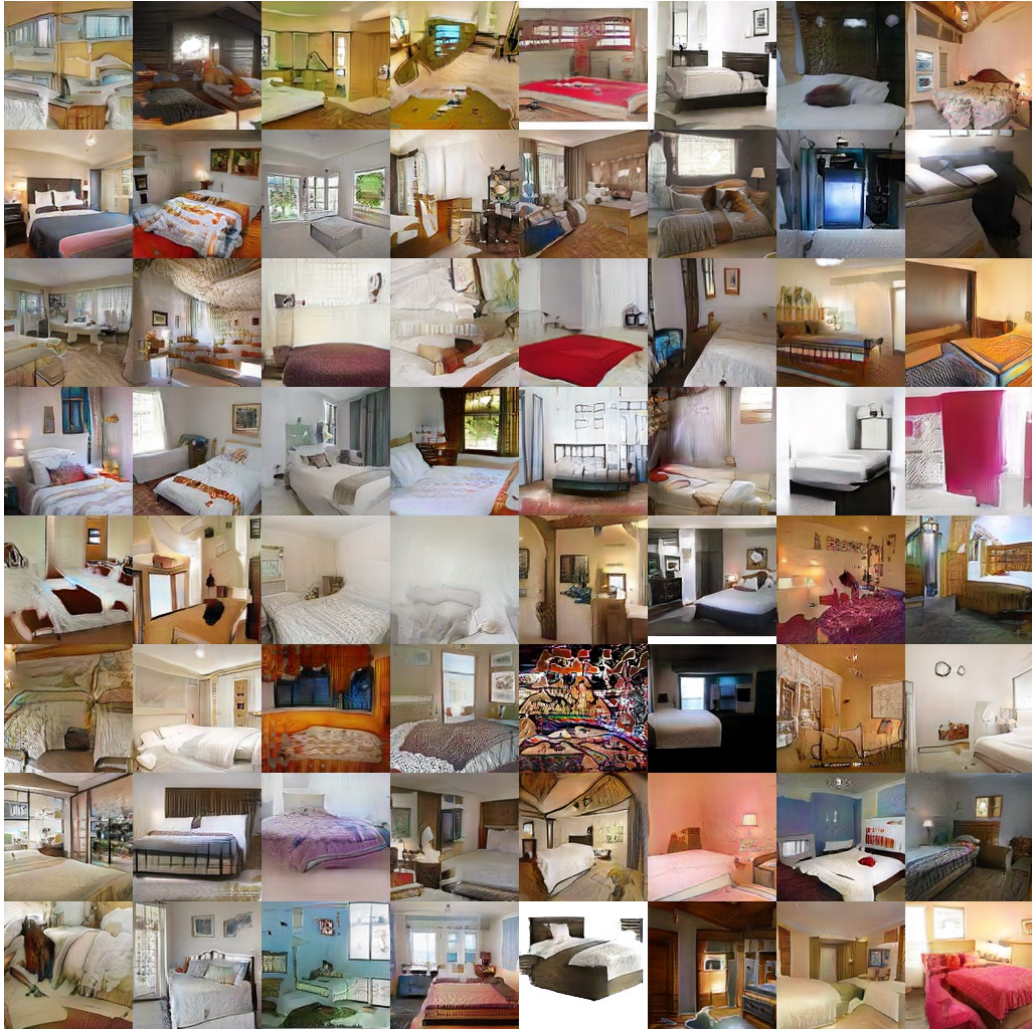


Figure 6: Samples of  $128 \times 128$  LSUN bedrooms. We believe these samples are at least comparable to the best published results so far.

In contrast, using our method we train a GAN which generates discrete outputs without resorting to complicated “backprop through discrete variable” methods, maximum-likelihood training, or fine-tuned architectures. We train a character-level GAN language model on the Google Billion Word dataset [6]. Our generator is a simple CNN which deterministically transforms a latent vector into a sequence of 32 one-hot character vectors through 1D convolutions. We apply a softmax nonlinearity at the output, but use no sampling step: during training, the softmax output is passed directly into the critic (which likewise, is a simple 1D CNN). When decoding samples, we just take the argmax of each output vector.

We present samples from the model in Table 1. To our knowledge this is the first reported result of a general language model trained entirely adversarially without a supervised maximum-likelihood loss. Our model makes frequent spelling errors (likely because it has to output each character independently) but nonetheless manages to learn quite a lot about the statistics of language. Although we do not claim that doing so is impossible, we were unable to produce comparable results with the standard GAN objective and a simple architecture.

The difference in performance between WGAN and other GANs can be explained by a simple fact. Consider the simplex  $\Delta_n = \{p \in \mathbb{R}^n : p_i \geq 0, \sum_i p_i = 1\}$ , and the set of vertices on the simplex (or one-hot vectors)  $V_n = \{p \in \mathbb{R}^n : p_i \in \{0, 1\}, \sum_i p_i = 1\} \subseteq \Delta_n$ . If we have a vocabulary of



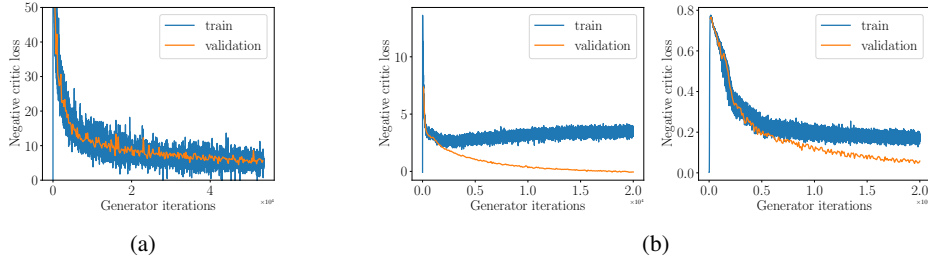


Figure 7: (a) The negative critic loss of our model on LSUN bedrooms converges toward a minimum as the network trains. (b) WGAN training and validation losses on a random 1000-digit subset of MNIST show overfitting when using either our method (left) or weight clipping (right). In particular, with our method, the critic overfits faster than the generator, causing the training loss to increase gradually over time even as the validation loss drops.

size  $n$  and we have a distribution  $\mathbb{P}_r$  over sequences of size  $T$ , we have that  $\mathbb{P}_r$  is a distribution on  $V_n^T = V_n \times \dots \times V_n$ . Since  $V_n^T$  is a subset of  $\Delta_n^T$ , we can also treat  $\mathbb{P}_r$  as a distribution on  $\Delta_n^T$  (by assigning zero probability mass to all points not in  $V_n^T$ ).

Now the interesting part is this:  $\mathbb{P}_r$  is discrete (or supported on a finite number of elements, namely  $V_n^T$ ) on  $\Delta_n^T$ , but  $\mathbb{P}_g$  can easily be a continuous distribution over  $\Delta_n^T$ . The KL divergences between two such distributions are infinite, and so the JS divergence is saturated. In practice, this means a discriminator will quickly learn to reject all samples that don't lie on  $V_n^T$  (sequences of one-hot vectors), and the gradients passed to the generator are meaningless. However, it is easily seen that the conditions of Theorem 1 and Corollary 1 of [2] are satisfied even on this non-standard learning scenario with  $\mathcal{X} = \Delta_n^T$ . This means that  $W(\mathbb{P}_r, \mathbb{P}_g)$  is still well defined, continuous everywhere and differentiable almost everywhere, and we can optimize it just like in any other continuous variable setting. The way this manifests is that in WGANs, the Lipschitz constraint forces the critic to provide a linear gradient from all  $\Delta_n^T$  towards towards the real points in  $V_n^T$ .

### 5.5 Meaningful loss curves and detecting overfitting

An important benefit of weight-clipped WGANs is that their loss correlates with sample quality and converges toward a minimum. To show that our method preserves this property, we train a WGAN with gradient penalty on the LSUN bedrooms dataset [30] and plot the negative of the critic's loss in Figure 7a. We see that the loss converges as the generator minimizes  $W(\mathbb{P}_r, \mathbb{P}_g)$ .

GANs, like all models trained on limited data, will eventually overfit, but the lack of a meaningful loss metric has made overfitting hard to detect in the past. To explore the loss curve's behavior when the network overfits, we train large unregularized WGANs on a random 1000-image subset of MNIST and plot the negative critic loss on both the training and validation sets in Figure 7b. In both WGAN and WGAN-GP, the two losses diverge, suggesting that the critic overfits and provides an inaccurate estimate of  $W(\mathbb{P}_r, \mathbb{P}_g)$ , at which point all bets are off regarding correlation with sample quality. However in WGAN with gradient penalty, the training loss gradually increases even while the validation loss drops.

[28] also measure overfitting in GANs by estimating the generator's log-likelihood. Compared to that work, our method detects overfitting in the critic (rather than the generator), is faster and easier to implement, and measures the same loss that the network minimizes.

## 6 Conclusion

In this work, we demonstrated problems with weight clipping in WGAN and introduced an alternative in the form of a penalty term in the critic loss which does not exhibit the same problems. Using our method, we demonstrated strong modeling performance and stability across a variety of architectures.

Now that we have a more stable algorithm for training GANs, we hope our work opens the path for stronger modeling performance on large-scale image datasets and language. Another interesting di-

rection is adapting our penalty term to the standard GAN objective function, where it might stabilize training by encouraging the discriminator to learn smoother decision boundaries.

## Acknowledgements

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## A Proof about the optimal WGAN critic

**Lemma 1.** Let  $\mathbb{P}_r$  and  $\mathbb{P}_g$  be two distributions in  $\mathcal{X}$ , a compact metric space. Then, there is a 1-Lipschitz function  $f^*$  which is the optimal solution of

$$\max_{\|f\|_L \leq 1} \mathbb{E}_{y \sim \mathbb{P}_r} [f(y)] - \mathbb{E}_{x \sim \mathbb{P}_g} [f(x)]$$

Let  $\pi$  be the optimal coupling between  $\mathbb{P}_r$  and  $\mathbb{P}_g$ , defined as the minimizer of:

$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\pi \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y) \sim \pi} [\|x - y\|]$$

Where  $\Pi(\mathbb{P}_r, \mathbb{P}_g)$  is the set of joint distributions  $\pi(x, y)$  whose marginals are  $\mathbb{P}_r$  and  $\mathbb{P}_g$ , respectively. Then, if  $f^*$  is differentiable<sup>2</sup>,  $\pi(x = y) = 0$ <sup>3</sup> and  $x_t = tx + (1 - t)y$  with  $0 \leq t \leq 1$ ,

$$\mathbb{P}_{(x,y) \sim \pi} \left[ \nabla f^*(x_t) = \frac{y - x_t}{\|y - x_t\|} \right] = 1$$

This, in particular, implies the norms of the gradients are 1.

*Proof.* Since  $\mathcal{X}$  is a compact space, by Theorem 5.10 of [24], part (iii), we know that there is an optimal  $f^*$ . By Theorem 5.10 of [24], part (ii) we know that if  $\pi$  is an optimal coupling,

$$\mathbb{P}_{(x,y) \sim \pi} [f^*(y) - f^*(x) = \|y - x\|] = 1$$

Let  $(x, y)$  be such that  $f^*(y) - f^*(x) = \|y - x\|$ . We can safely assume that  $x \neq y$  as well, since this happens under  $\pi$  with probability 1. Let  $\psi(t) = f^*(x_t) - f^*(x)$ . We claim that  $\psi(t) = \|x_t - x\| = t\|y - x\|$ .

Let  $t, t' \in [0, 1]$ , then

$$\begin{aligned} |\psi(t) - \psi(t')| &= \|f^*(x_t) - f^*(x_{t'})\| \\ &\leq \|x_t - x_{t'}\| \\ &= |t - t'| \|x - y\| \end{aligned}$$

Therefore,  $\psi$  is  $\|x - y\|$ -Lipschitz. This in turn implies

$$\begin{aligned} \psi(1) - \psi(0) &= \psi(1) - \psi(t) + \psi(t) - \psi(0) \\ &\leq (1 - t)\|x - y\| + \psi(t) - \psi(0) \\ &\leq (1 - t)\|x - y\| + t\|x - y\| \\ &= \|x - y\| \end{aligned}$$

However,  $|\psi(1) - \psi(0)| = |f^*(y) - f^*(x)| = \|y - x\|$  so the inequalities have to actually be equalities. In particular,  $\psi(t) - \psi(0) = t\|x - y\|$ , and  $\psi(0) = f^*(x) - f^*(x) = 0$ . Therefore,  $\psi(t) = t\|x - y\|$  and we finish our claim.

Let

$$\begin{aligned} v &= \frac{y - x_t}{\|y - x_t\|} \\ &= \frac{y - ((1 - t)x - ty)}{\|y - ((1 - t)x - ty)\|} \\ &= \frac{(1 - t)(y - x)}{\|(1 - t)\|y - x\|} \\ &= \frac{y - x}{\|y - x\|} \end{aligned}$$

<sup>2</sup>We can actually assume much less, and talk only about directional derivatives on the direction of the line; which we show in the proof always exist. This would imply that in every point where  $f^*$  is differentiable (and thus we can take gradients in a neural network setting) the statement holds.

<sup>3</sup>This assumption is in order to exclude the case when the matching point of sample  $x$  is  $x$  itself. It is satisfied in the case that  $\mathbb{P}_r$  and  $\mathbb{P}_g$  have supports that intersect in a set of measure 0, such as when they are supported by two low dimensional manifolds that don't perfectly align [1].

Now we know that  $f^*(x_t) - f^*(x) = \psi(t) = t\|x - y\|$ , so  $f^*(x_t) = f^*(x) + t\|x - y\|$ . Then, we have the partial derivative

$$\begin{aligned}
\frac{\partial}{\partial v} f^*(x_t) &= \lim_{h \rightarrow 0} \frac{f^*(x_t + hv) - f^*(x_t)}{h} \\
&= \lim_{h \rightarrow 0} \frac{f^*\left(x + t(y - x) + \frac{h}{\|y - x\|}(y - x)\right) - f^*(x_t)}{h} \\
&= \lim_{h \rightarrow 0} \frac{f^*\left(x_t + \frac{h}{\|y - x\|}(y - x)\right) - f^*(x_t)}{h} \\
&= \lim_{h \rightarrow 0} \frac{f^*(x) + \left(t + \frac{h}{\|y - x\|}\right)\|x - y\| - (f^*(x) + t\|x - y\|)}{h} \\
&= \lim_{h \rightarrow 0} \frac{h}{h} \\
&= 1
\end{aligned}$$

If  $f^*$  is differentiable at  $x_t$ , we know that  $\|\nabla f^*(x_t)\| \leq 1$  since it is a 1-Lipschitz function. Therefore, by simple Pythagoras and using that  $v$  is a unit vector

$$\begin{aligned}
1 &\leq \|\nabla f^*(x_t)\|^2 \\
&= \langle v, \nabla f^*(x_t) \rangle^2 + \|\nabla f^*(x_t) - \langle v, \nabla f^*(x_t) \rangle v\|^2 \\
&= \left| \frac{\partial}{\partial v} f^*(x_t) \right|^2 + \|\nabla f^*(x_t) - v \frac{\partial}{\partial v} f^*(x_t)\|^2 \\
&= 1 + \|\nabla f^*(x_t) - v\|^2 \\
&\leq 1
\end{aligned}$$

The fact that both extremes of the inequality coincide means that it was all an equality and  $1 = 1 + \|\nabla f^*(x_t) - v\|^2$  so  $\|\nabla f^*(x_t) - v\| = 0$  and therefore  $\nabla f^*(x_t) = v$ . This shows that  $\nabla f^*(x_t) = \frac{y - x_t}{\|y - x_t\|}$ .

To conclude, we showed that if  $(x, y)$  have the property that  $f^*(y) - f^*(x) = \|y - x\|$ , then  $\nabla f^*(x_t) = \frac{y - x_t}{\|y - x_t\|}$ . Since this happens with probability 1 under  $\pi$ , we know that

$$\mathbb{P}_{(x,y) \sim \pi} \left[ \nabla f^*(x_t) = \frac{y - x_t}{\|y - x_t\|} \right] = 1$$

and we finished the proof. □

## B Hyperparameters used for stability experiments

- WGAN with gradient penalty: Adam ( $\alpha = .0001, \beta_1 = .5, \beta_2 = .9$ )
- WGAN with weight clipping: RMSProp ( $\alpha = .00005$ )
- DCGAN: Adam ( $\alpha = .0002, \beta_1 = .5$ )
- LSGAN: RMSProp ( $\alpha = .0001$ ) [chosen by search over  $\alpha = .001, .0002, .0001$ ]

## C CIFAR-10 ResNet architecture

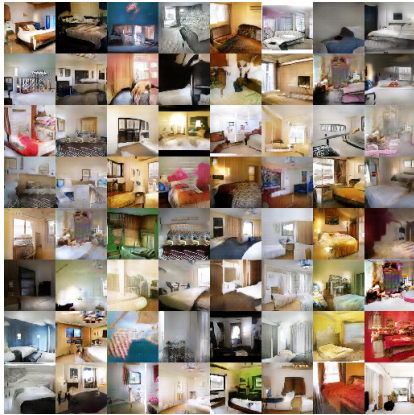
The generator and critic are residual networks; we use pre-activation residual blocks with two  $3 \times 3$  convolutional layers each and ReLU nonlinearity. Some residual blocks perform downsampling (in the critic) using mean pooling after the second convolution, or nearest-neighbor upsampling (in the generator) before the second convolution. We use batch normalization in the generator but not the critic. We optimize using Adam with learning rate  $2 \times 10^{-4}$ , decayed linearly to 0 over 100K generator iterations, and batch size 64.

For further architectural details, please refer to our open-source implementation.

Generator $G(z)$			
	Kernel size	Resample	Output shape
$z$	-	-	128
Linear	-	-	$128 \times 4 \times 4$
Residual block	$[3 \times 3] \times 2$	Up	$128 \times 8 \times 8$
Residual block	$[3 \times 3] \times 2$	Up	$128 \times 16 \times 16$
Residual block	$[3 \times 3] \times 2$	Up	$128 \times 32 \times 32$
Conv, $\tanh$	$3 \times 3$	-	$3 \times 32 \times 32$

Critic $D(x)$			
	Kernel size	Resample	Output shape
Residual block	$[3 \times 3] \times 2$	Down	$128 \times 16 \times 16$
Residual block	$[3 \times 3] \times 2$	Down	$128 \times 8 \times 8$
Residual block	$[3 \times 3] \times 2$	-	$128 \times 8 \times 8$
Residual block	$[3 \times 3] \times 2$	-	$128 \times 8 \times 8$
ReLU, mean pool	-	-	128
Linear	-	-	1

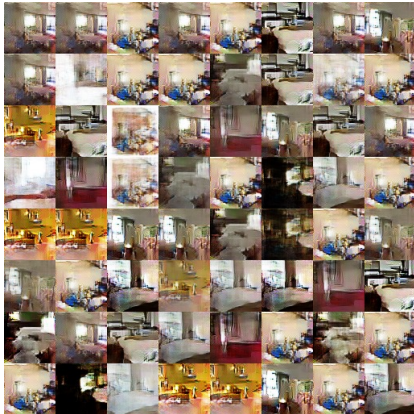
## D More LSUN samples



Method: DCGAN  
 $G$ : DCGAN,  $D$ : DCGAN



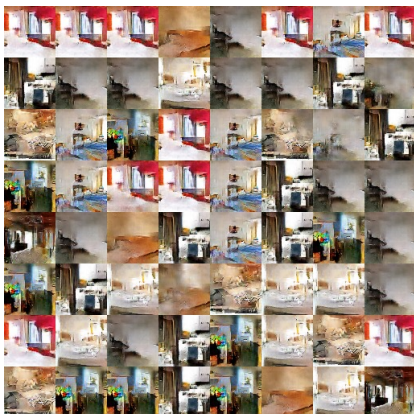
Method: DCGAN  
 $G$ : No BN and const. filter count



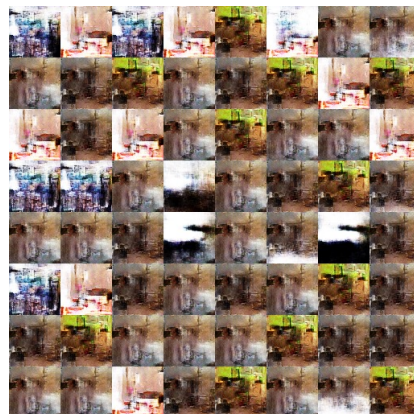
Method: DCGAN  
 $G$ : 4-layer 512-dim ReLU MLP



Method: DCGAN  
No normalization in either  $G$  or  $D$



Method: DCGAN  
Gated multiplicative nonlinearities



Method: DCGAN  
 $\tanh$  nonlinearities



Method: DCGAN  
101-layer ResNet  $G$  and  $D$



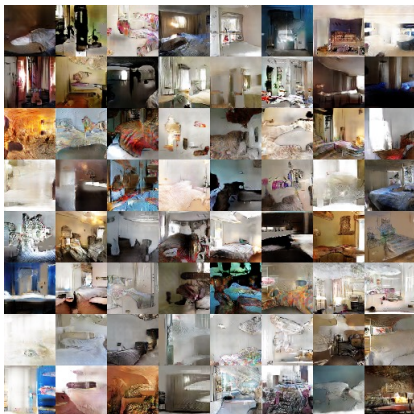
Method: LSGAN  
 $G$ : DCGAN,  $D$ : DCGAN



Method: LSGAN  
 $G$ : No BN and const. filter count



Method: LSGAN  
 $G$ : 4-layer 512-dim ReLU MLP



Method: LSGAN  
No normalization in either  $G$  or  $D$

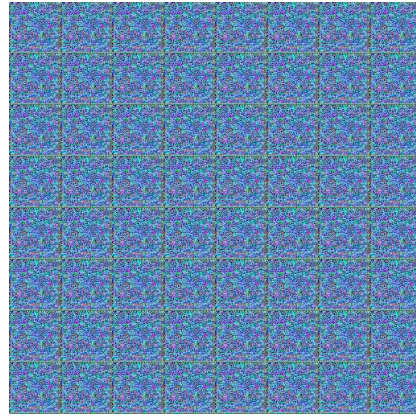


Method: LSGAN  
Gated multiplicative nonlinearities

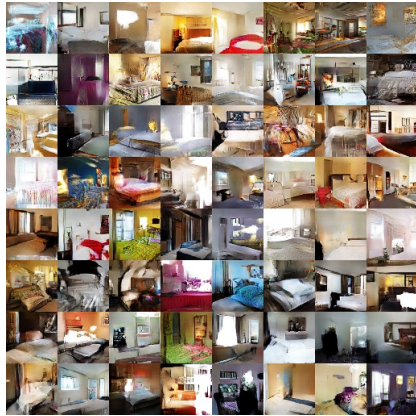




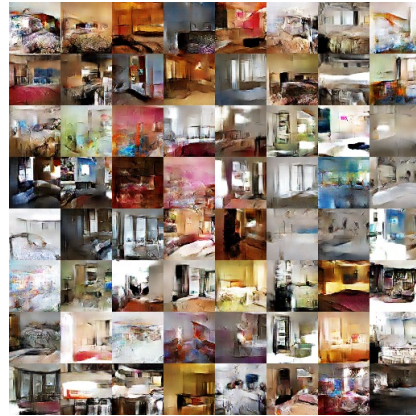
Method: LSGAN  
*tanh* nonlinearities



Method: LSGAN  
101-layer ResNet *G* and *D*



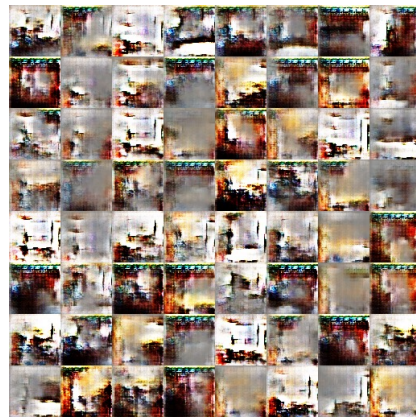
Method: WGAN with clipping  
*G*: DCGAN, *D*: DCGAN



Method: WGAN with clipping  
*G*: No BN and const. filter count



Method: WGAN with clipping  
*G*: 4-layer 512-dim ReLU MLP



Method: WGAN with clipping  
No normalization in either *G* or *D*



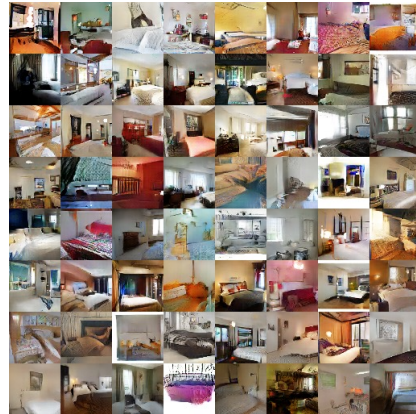
Method: WGAN with clipping  
Gated multiplicative nonlinearities



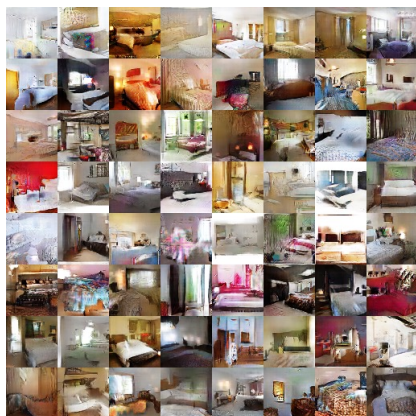
Method: WGAN with clipping  
*tanh* nonlinearities



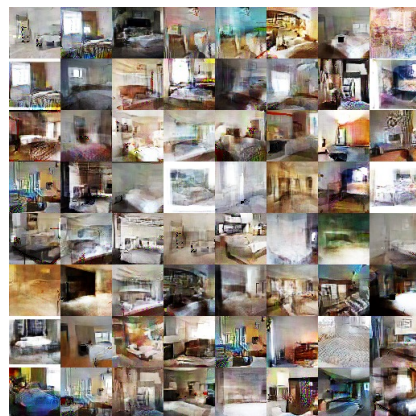
Method: WGAN with clipping  
101-layer ResNet  $G$  and  $D$



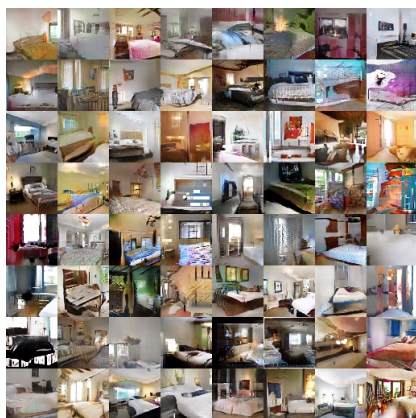
Method: WGAN-GP (ours)  
 $G$ : DCGAN,  $D$ : DCGAN



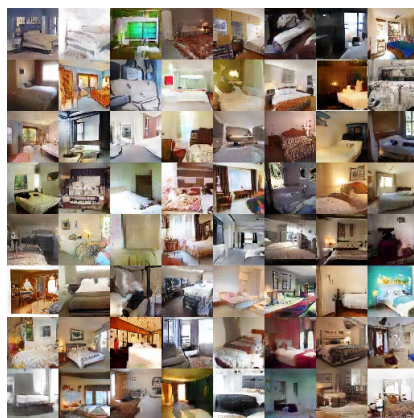
Method: WGAN-GP (ours)  
 $G$ : No BN and const. filter count



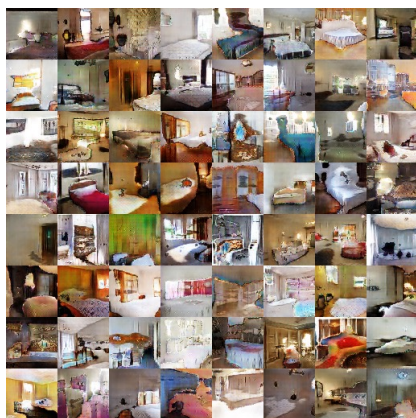
Method: WGAN-GP (ours)  
 $G$ : 4-layer 512-dim ReLU MLP



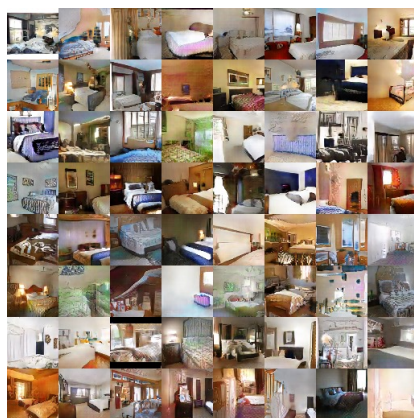
Method: WGAN-GP (ours)  
No normalization in either  $G$  or  $D$



Method: WGAN-GP (ours)  
Gated multiplicative nonlinearities



Method: WGAN-GP (ours)  
 $\tanh$  nonlinearities



Method: WGAN-GP (ours)  
101-layer ResNet  $G$  and  $D$